Estimating labor market power from job applications

ONLINE APPENDIX

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Contents

A Worked example

Let the choice set be $\mathcal{J} = \{a, b, c\}$ and normalize $\delta_a = 0$, so that a is our base category providing the necessary location normalization for identification. Table A.1 below summarizes the relevant sets that appear on the left-hand size of Equation (A.1) defining the conditional inclusion probability $\mathbb{P}(j \in A_i \mid n_i = n)$ as a function of mean utilities $\boldsymbol{\delta}$. Here, $J = 3$, so the outer summation index k ranges from $J - n$ to $J - 1$ for $n \in \{1, 2, 3\}$.

$$s_{j|n}(\boldsymbol{\delta}) = \sum_{k=J-n}^{J-1} \sum_{A \in \mathcal{R}_k(\mathcal{B}_j)} \frac{\exp(\delta_j)}{\sum\limits_{\ell \in \{j\} \cup A} \exp(\delta_\ell)} + \sum_{s=1}^{|\mathcal{B}_j \setminus A|} (-1)^s \sum_{B \in \mathcal{R}_s(\mathcal{B}_j \setminus A)} \frac{\exp(\delta_j)}{\sum\limits_{\ell \in \{j\} \cup A \cup B} \exp(\delta_\ell)} \tag{A.1}$$

Note that the outer summation defines a recursive structure over n: The term corresponding to $k = J - 1$ appears in the outer summation for all $n \in \{1, \ldots, J\}$, the term corresponding to $k = J - 2$ appears in the outer summation for every $n \in \{2, \ldots, J\}$, and so on until the term for $k = 1$ appears in the summations for $n \in \{J - 1, J\}$, and the term for $k = 0$ appears only in the summation for $n = J$. This is evident in the structure of Table A.1. We can use this recursive structure —together with the fact that $|\mathcal{B}_j \setminus A| = (J-1) - (J-n) = n-1$ for every $A \subseteq \mathcal{B}_j$ such that $|A| = J - n$— to rewrite (A.1) as the recursion

$$s_{j|n}(\boldsymbol{\delta}) = \sum_{A \in \mathcal{R}_{J-n}(\mathcal{B}_j)} \left[\frac{\exp(\delta_j)}{\sum_{\ell \in \{j\} \cup A} \exp(\delta_\ell)} + \sum_{s=1}^{n-1} (-1)^s \sum_{B \in \mathcal{R}_s(\mathcal{B}_j \setminus A)} \frac{\exp(\delta_j)}{\sum_{\ell \in \{j\} \cup A \cup B} \exp(\delta_\ell)} \right] + s_{j|n-1}(\boldsymbol{\delta}) \quad \text{(A.2)}$$

for $n \in \{2, \dots, J\}$.

Table A.1. Underlying indices and sets defining $s_{j|n}(\boldsymbol{\delta})$

j	$\mathcal{B}_j$	n	k	$\mathcal{R}_k(\mathcal{B}_j)$	A	$\mathcal{B}_j \setminus A$	$\|\mathcal{B}_j \setminus A\|$	s	$\mathcal{R}_s(\mathcal{B}_j \setminus A)$	B
a	$\{b,c\}$	1	2	$\{\{b,c\}\}$	$\{b,c\}$	$\emptyset$	0	—	—	—
		2	1	$\{\{b\},\{c\}\}$	$\{b\}$	$\{c\}$	1	1	$\{\{c\}\}$	$\{c\}$
					$\{c\}$	$\{b\}$	1	1	$\{\{b\}\}$	$\{b\}$
			2	$\{\{b,c\}\}$	$\{b,c\}$	$\emptyset$	0	—	—	—
		3	0	$\{\emptyset\}$	$\emptyset$	$\{b,c\}$	2	1	$\{\{b\},\{c\}\}$	$\{b\}$ $\{c\}$
								2	$\{\{b,c\}\}$	$\{b,c\}$
			1	$\{\{b\},\{c\}\}$	$\{b\}$	$\{c\}$	1	1	$\{\{c\}\}$	$\{c\}$
					$\{c\}$	$\{b\}$	1	1	$\{\{b\}\}$	$\{b\}$
			2	$\{\{b,c\}\}$	$\{b,c\}$	$\emptyset$	0	—	—	—

Notes: Combinations of the relevant sets and indices for each term on the right-hand side of Equation (A.1) for our $\mathcal{J} = \{a,b,c\}$ example.

Start $n = 1$ and arbitrary $j \in \{a,b,c\}$. Equation (A.1) yields

$$s_{j|1}(\boldsymbol{\delta}) = \sum_{k=3-1}^{3-1} \sum_{A \in \mathcal{R}_k(\mathcal{B}_j)} \frac{\exp(\delta_j)}{\sum_{\ell \in \{j\} \cup A} \exp(\delta_\ell)} + \sum_{s=1}^{|\mathcal{B}_j \setminus A|} (-1)^s \sum_{B \in \mathcal{R}_s(\mathcal{B}_j \setminus A)} \frac{\exp(\delta_j)}{\sum_{\ell \in \{j\} \cup A \cup B} \exp(\delta_\ell)}$$

$$= \sum_{A \in \mathcal{R}_2(\mathcal{B}_j)} \frac{\exp(\delta_j)}{\sum_{\ell \in \{j\} \cup A} \exp(\delta_\ell)}$$

$$= \frac{\exp(\delta_j)}{\sum_{\ell \in \{a,b,c\}} \exp(\delta_\ell)}, \tag{A.3}$$

where the second equality follows since $\mathcal{R}_2(\mathcal{B}_j) = \{\mathcal{B}_j\} \implies A = \mathcal{B}_j \implies \mathcal{B}_j \setminus A = \emptyset$, and the last equality follows from $\{j\} \cup \mathcal{B}_j = \mathcal{J}$.

It is fairly easy to see that (δ_b, δ_c) are identified by the inclusion probabilities conditional on $n_i = 1$ and the location normalization $\delta_a = 0$:

$$\begin{aligned} \ln\big(s_{b|1}(\boldsymbol{\delta})\big) - \ln\big(s_{a|1}(\boldsymbol{\delta})\big) &= \ln\big(\exp(\delta_b)\big) - \ln\big(1 + \exp(\delta_b) + \exp(\delta_c)\big) + \ln\big(1 + \exp(\delta_b) + \exp(\delta_c)\big) \\ &= \delta_b \end{aligned} \tag{A.4}$$

$$\begin{aligned} \ln\big(s_{c|1}(\boldsymbol{\delta})\big) - \ln\big(s_{a|1}(\boldsymbol{\delta})\big) &= \ln\big(\exp(\delta_c)\big) - \ln\big(1 + \exp(\delta_b) + \exp(\delta_c)\big) + \ln\big(1 + \exp(\delta_b) + \exp(\delta_c)\big) \\ &= \delta_c. \end{aligned} \tag{A.5}$$

Now, use Equation (A.1) to obtain $s_{j|2}(\boldsymbol{\delta})$ for each $j \in \{a, b, c\}$ using Equation (A.3):

$$\begin{aligned} s_{a|2}(\boldsymbol{\delta}) &= \sum_{A \in \mathcal{R}_1(\mathcal{B}_a)} \left[\frac{\exp(\delta_a)}{\sum_{\ell \in \{a\} \cup A} \exp(\delta_\ell)} + \sum_{s=1}^{1} (-1)^s \sum_{B \in \mathcal{R}_s(\mathcal{B}_a \setminus A)} \frac{\exp(\delta_a)}{\sum_{\ell \in \{j\} \cup A \cup B} \exp(\delta_\ell)} \right] + s_{a|1}(\boldsymbol{\delta}) \\ &= \Bigg[\underbrace{\frac{\exp(\delta_a)}{\sum_{\ell \in \{a\} \cup \{b\}} \exp(\delta_\ell)} - \underbrace{\frac{\exp(\delta_a)}{\sum_{\ell \in \{a\} \cup \{b\} \{c\}} \exp(\delta_\ell)}}_{B=\{c\}}}_{A=\{b\}} + \underbrace{\frac{\exp(\delta_a)}{\sum_{\ell \in \{a\} \cup \{c\}} \exp(\delta_\ell)} - \underbrace{\frac{\exp(\delta_a)}{\sum_{\ell \in \{a\} \cup \{b\} \{c\}} \exp(\delta_\ell)}}_{B=\{b\}}}_{A=\{c\}} \Bigg] + s_{a|1}(\boldsymbol{\delta}) \\ &= \frac{\exp(\delta_a)}{\exp(\delta_a) + \exp(\delta_b)} + \frac{\exp(\delta_a)}{\exp(\delta_a) + \exp(\delta_c)} - \frac{\exp(\delta_a)}{\exp(\delta_a) + \exp(\delta_b) + \exp(\delta_c)}, \end{aligned} \tag{A.6}$$

where the first equality follows from $n = 2$ and $J = 3 \implies J - n = 1$ and $n - 1 = 1$, and the last equality uses Equation (A.3). Similar algebra yields

$$s_{b|2}(\boldsymbol{\delta}) = \frac{\exp(\delta_b)}{\exp(\delta_a) + \exp(\delta_b)} + \frac{\exp(\delta_b)}{\exp(\delta_b) + \exp(\delta_c)} - \frac{\exp(\delta_b)}{\exp(\delta_a) + \exp(\delta_b) + \exp(\delta_c)} \tag{A.7}$$

$$s_{c|2}(\boldsymbol{\delta}) = \frac{\exp(\delta_c)}{\exp(\delta_a) + \exp(\delta_c)} + \frac{\exp(\delta_c)}{\exp(\delta_b) + \exp(\delta_c)} - \frac{\exp(\delta_c)}{\exp(\delta_a) + \exp(\delta_b) + \exp(\delta_c)} \tag{A.8}$$

Worked Example: Local Invertibility for $J = 3$, $n = 2$

This example verifies the nonsingularity of the reduced Jacobian in Proposition **??** for the simplest nontrivial case, with three alternatives ($J = 3$) and portfolio size $n = 2$. The calculation illustrates the structure of the Jacobian and confirms that the conditions for local identification are satisfied globally.

Lemma 1 (Jacobian nonsingularity for $J = 3, n = 2$)**.** *Let $\mathcal{J} = \{a, b, c\}$ and normalize $\delta_a = 0$. Write $x = \exp(\delta_b)$ and $y = \exp(\delta_c)$. Under the EV_1 assumption, the conditional inclusion probabilities for portfolios of size two are*

$$s_{a|2} = \frac{x + y}{xy + x + y}, \qquad s_{b|2} = \frac{xy + x}{xy + x + y}, \qquad s_{c|2} = \frac{xy + y}{xy + x + y}, \tag{A.9}$$

where $s_{a|2} + s_{b|2} + s_{c|2} = 2$ as required.

The reduced mapping $f_2 : (\delta_b, \delta_c) \mapsto (s_{b|2}, s_{c|2})$ has Jacobian

$$Df_2(\delta_b, \delta_c) = \frac{1}{(xy + x + y)^2} \begin{pmatrix} x\,y(y+1) & -x\,y \\ -x\,y & x\,y(x+1) \end{pmatrix}. \tag{A.10}$$

The determinant of this Jacobian is

$$\det Df_2 = \frac{x^2y^2}{(xy + x + y)^4}\big[(y+1)(x+1) - 1\big] = \frac{x^2y^2(xy + x + y)}{(xy + x + y)^4} > 0 \quad \forall\, x, y > 0. \tag{A.11}$$

Hence $Df_2(\delta_b, \delta_c)$ is nonsingular everywhere, with positive diagonal entries, negative off-diagonals, and zero row sums. The sign pattern confirms that Df_2 is an M-matrix, implying global injectivity of f_2 up to the location normalization $\delta_a = 0$.

Proof. The derivatives follow by direct differentiation of the expressions in (D.15). The positive definiteness of the principal minors implies full rank of the reduced Jacobian. Because each diagonal element $\partial s_{j|2}/\partial \delta_j$ is positive, each off-diagonal $\partial s_{j|2}/\partial \delta_\ell$ ($\ell \neq j$) is negative, and the row-sum identity $\sum_j \partial s_{j|2}/\partial \delta_\ell = 0$ holds, the Jacobian satisfies the sufficient conditions for nonsingularity of an M-matrix. □

Interpretation. This explicit case confirms that for interior portfolio sizes, the mapping $\boldsymbol{\delta} \mapsto \{s_{j|n}(\boldsymbol{\delta})\}_{j=1}^{J}$ is locally one-to-one (up to location). For $n = 3$ (the full portfolio), $s_{j|3} \equiv 1$ and the Jacobian collapses to zero, while for $n = 1$ the mapping reduces to the standard multinomial logit shares for which the same local invertibility result is well known. Hence identification of $\boldsymbol{\delta}$ requires that at least one interior n ($1 < n < J$) has positive population probability mass, as stated in Assumption **??**.

B Further data detalis

Table B.1. Job seeker variables

Characteristic	Variable type	Description
Job seeker identifier	Numerical	Anonymized ID
CV registration date	Numerical	Daily date
CV modification date	Numerical	Daily date Last modification as of `12aug2020`
Availability to work	Binary	As of last CV update
Salary expectation	Numerical	Amount in CLP $
Salary expectation disclosure preference	Binary	As of last CV update
Date of birth	Numerical	Daily date
Sex	Categorical	Male, female, prefer not to say
Nationality	Text string	Unstructured
Marital status	Categorical	As of last CV update
Region of residence	Categorical	As of last CV update
City of residence	Categorical	As of last CV update
District of residence	Categorical	As of last CV update
Education level	Categorical	Highest degree
Last study programs	Text string	Structured: Program, institution, status Up to three
Employment status	Binary	As of last CV update
Experience	Numeric	Years of work experience as of last CV update
Last job	Text string	Structured: Start and ending dates, job title, and monthly salary

Table B.2. Job ad variables

Characteristic	Variable type	Description
Job ad identifier	Numerical ID	Anonymized
Firm identifier	Numerical	Anonymized ID
Publication date	Numerical	Daily date
Expiry date	Numerical	Daily date
Paid advertisement	Binary	Paid, free
Number of vacancies	Numerical	
Expected monthly salary	Numerical	Amount in CLP $
Salary disclosure setting	Binary	Public, hidden
Job title	Text string	Free format
Advertisement	Text string	Unstructured, free-format
Job field	Categorical	175 fields
Employer economic activity	Categorical	68 activities
Place of work	Text string	Unstructured, free-format
Job requisites	Text string	Unstructured, free-format
Contract duration	Text string	Unstructured, free-format
Work arrangement	Categorical	10 arrangements
Work experience requirement	Numerical	Years of experience
Education level requirement	Text string	Unstructured, free-format
Study situation requirement	Categorical	5 situations
Specific degree requirement	Categorical	6 degrees
Computer skills requirement	Categorical	7 skill levels

Table B.3. Employer variables

Characteristic	Variable type	Description
Firm identifier	Numerical	Anonymized ID
Firm name	Text string	Free format
Economic activity	Categorical	68 activities
Region	Categorical	16 regions
City	Text string	Free format
Number of employees	Categorical	8 size bins

Table B.4. Applications variables

Characteristic	Variable type	
Job ad identifier	Numerical	Anonymized ID
Job seeker identifier	Numerical	Anonymized ID
Application date	Numerical	Daily date