EC317: Problem Set 4 Notes

1. Consider a person who can work up to 80 hours each week at a pre-tax wage of \$20 per hour but faces a constant 20% payroll tax. Under these conditions, the worker maximizes her utility by choosing to work 50 hours each week. The government proposes a negative income tax whereby everyone is given \$300 each week and anyone can supplement her income further by working. To pay for the negative income tax, the payroll tax rate will be increased to 50%.

   (a) On a single graph, draw the worker's original budget line and her budget line under the negative income tax.

   Answer:

   Let $t \in 0, 1$ index the original —i.e., without the negative income tax— and new —i.e., with negative income tax— situations. The budget constraint in situation t can be obtained by combining the following equations

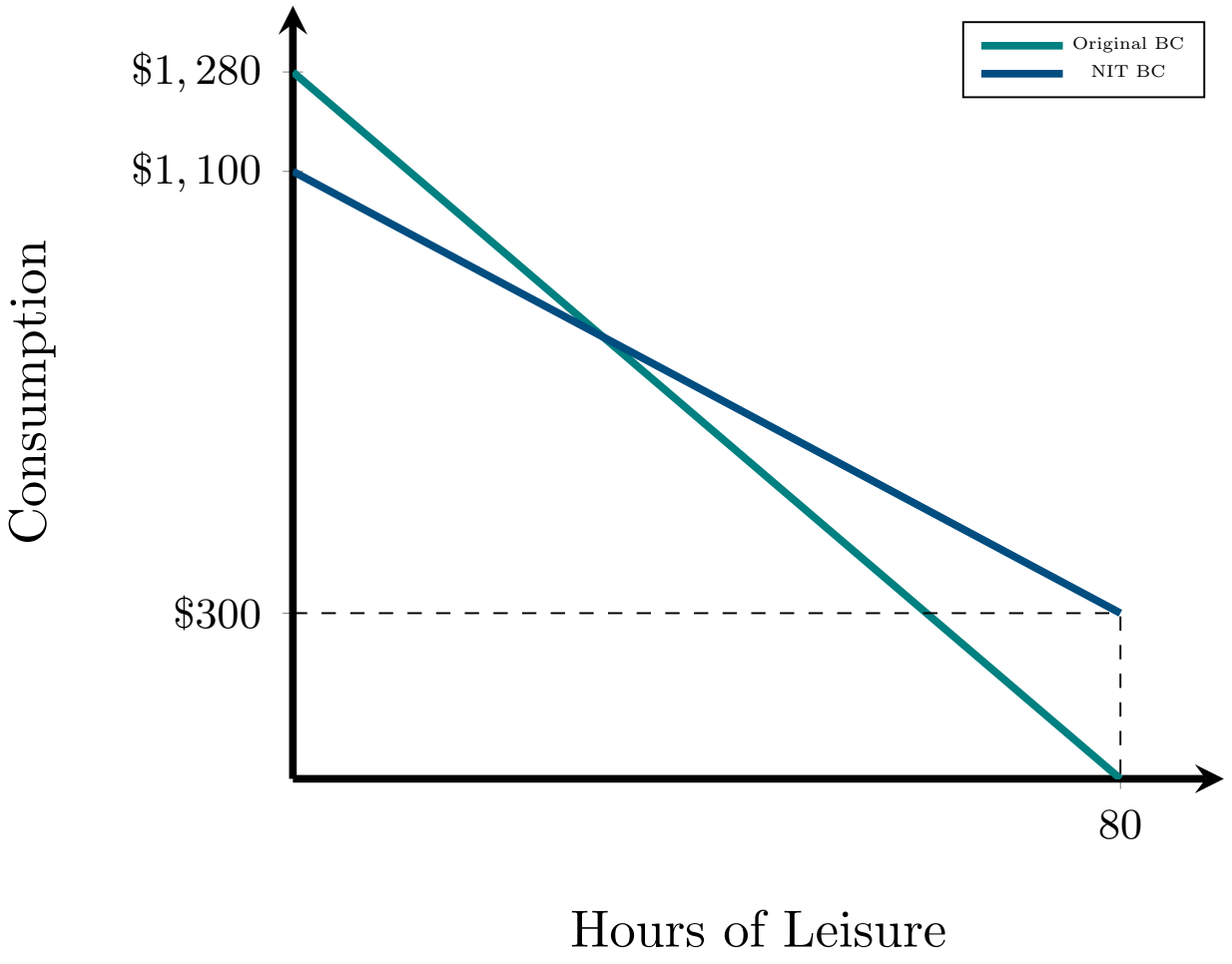
$$\begin{aligned} C &= V_t + (1 - \tau_t)\, w\, H \\ H + L &= T \\ \implies C + (1 - \tau_t)\, w\, L &= V_t + (1 - \tau_t)\, w\, T. \end{aligned}$$

   Plugging in the parameter values —$V_0 = 0$, $V_1 = 300$, $w = 20$, $\tau_0 = 0.2$,$\tau_1 = 0.5$, $T = 80$— we obtain

$$\begin{aligned} \text{Original BC: } C &= 1,280 - 16\, L \\ \text{NIT BC: } C &= 1,100 - 10\, L, \end{aligned}$$

   Which are plotted in Figure 1 below.

Figure 1: Change in budget line



(b) Show that the worker will choose to work fewer hours if the negative income tax is adopted.

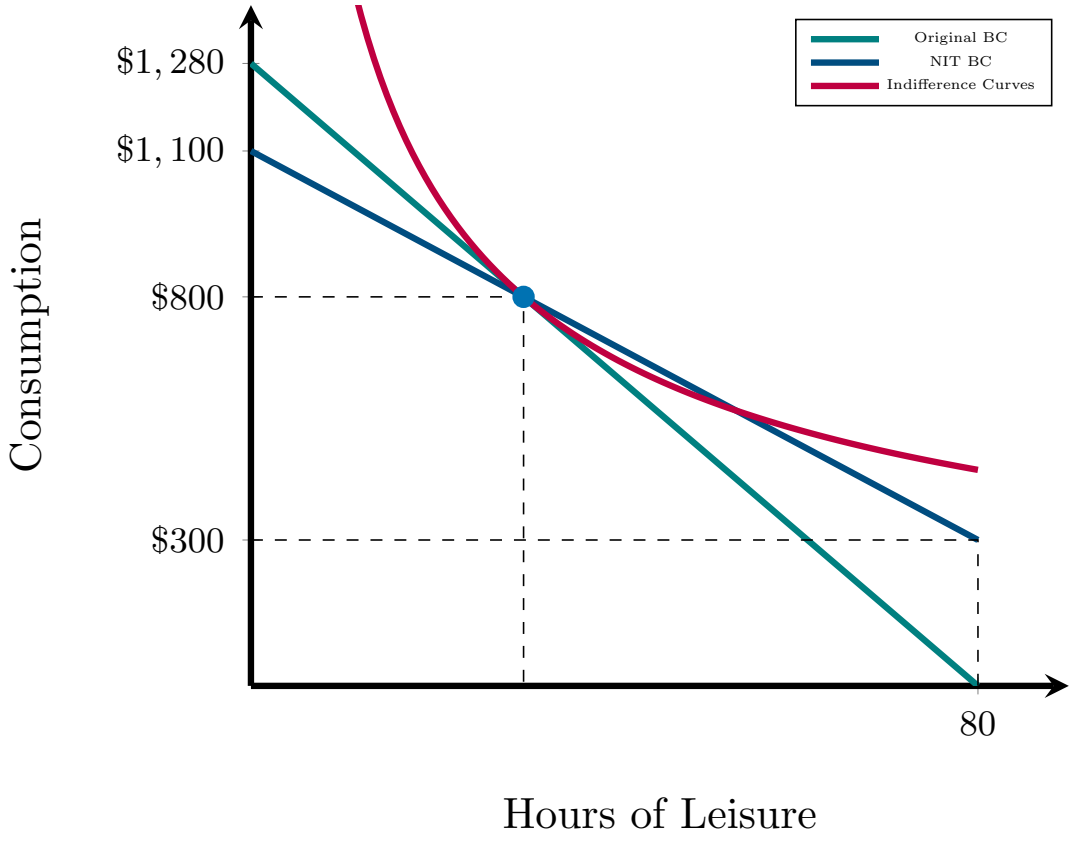
Answer:

We know that the worker initially works 50 hours, i.e., $H_0 = 50 \iff L_0 = 80 - 50 = 30$. Notice that the budget lines intersect exactly at $L = 30$:

$$1,280 - 16\,L = 1,100 - 10\,L$$
$$\iff L = 30.$$

Therefore, the worker's indifference curve through the original optimal bundle must be below the new budget line to the right of $L = 30$. Moving in that direction along the new budget line increases utility, so the new optimal bundle must be to the right of the original bundle. Hence $L_1 > L_0 = 30 \implies H_1 < H_0 = 50$.

Figure 2: The worker works fewer hours under NIT



(c) Will the worker's utility be greater under the negative income tax?

Answer:

In this particular case, the worker's utility will increase under the negative income tax because she could continue to leisure 30 hours each week and receive \$800 (which was her outcome before the negative income tax) but instead the worker decides to leisure more (and consume less). This change in behavior must increase her utility.

2. US Social Security retirement benefits are determined by earnings previous to the retirement decision. However, the base amount of benefits will be reduced for retirees who continue to do some work. This is called the Social security earnings test. The rules are (roughly) as follows: Retirees can earn up to \$5,000 (the earnings test floor) annually and still get the full benefit. For earnings above this amount, retirement benefits are reduced by 50 cents per additional dollar earned until the individual receives no more benefits.

   (a) Draw the budget constraint in terms of annual hours for a retiree who can earn \$20 an hour and receives \$10,000 in Social Security benefits a year before the earnings test kicks in. Also draw the budget constraint for the same worker if there was no earnings test. Carefully label any kinks in your graph.

   Answer:

   Let b represent yearly social security benefits before the earnings test kicks in. Under the earnings test, nonlabour income of a retiree is a function of their current yearly earnings wH:

$$V(wH) = \begin{cases} b & \text{if } \; wH \leq 5,000 \\ b - 0.5\,(wH - 5,000) & \text{if } \; 5,000 < wH \leq 2b + 5,000 \\ 0 & \text{if } \; wH > 2b + 5,000, \end{cases}$$

   since social security benefits run out when $b - 0.5\,(wH - 5,000) = 0 \iff wH = 2b + 5,000$. Plugging in the parameter values —$w = 20$ and $b = 10,000$—, we obtain the following budget constraint:

$$V(wH) = \begin{cases} 10,000 & \text{if } \; 20H \in [0,\, 5,000] \iff H \in [0,\, 250] \\ 12,500 - 10H & \text{if } \; 20H \in (5,000,\, 25,000] \iff H \in (250,\, 1,250] \\ 0 & \text{if } \; 20H \in (25,000,\, 20\,T] \iff H \in (1,250,\, T]\,, \end{cases}$$

   This leads to budget line

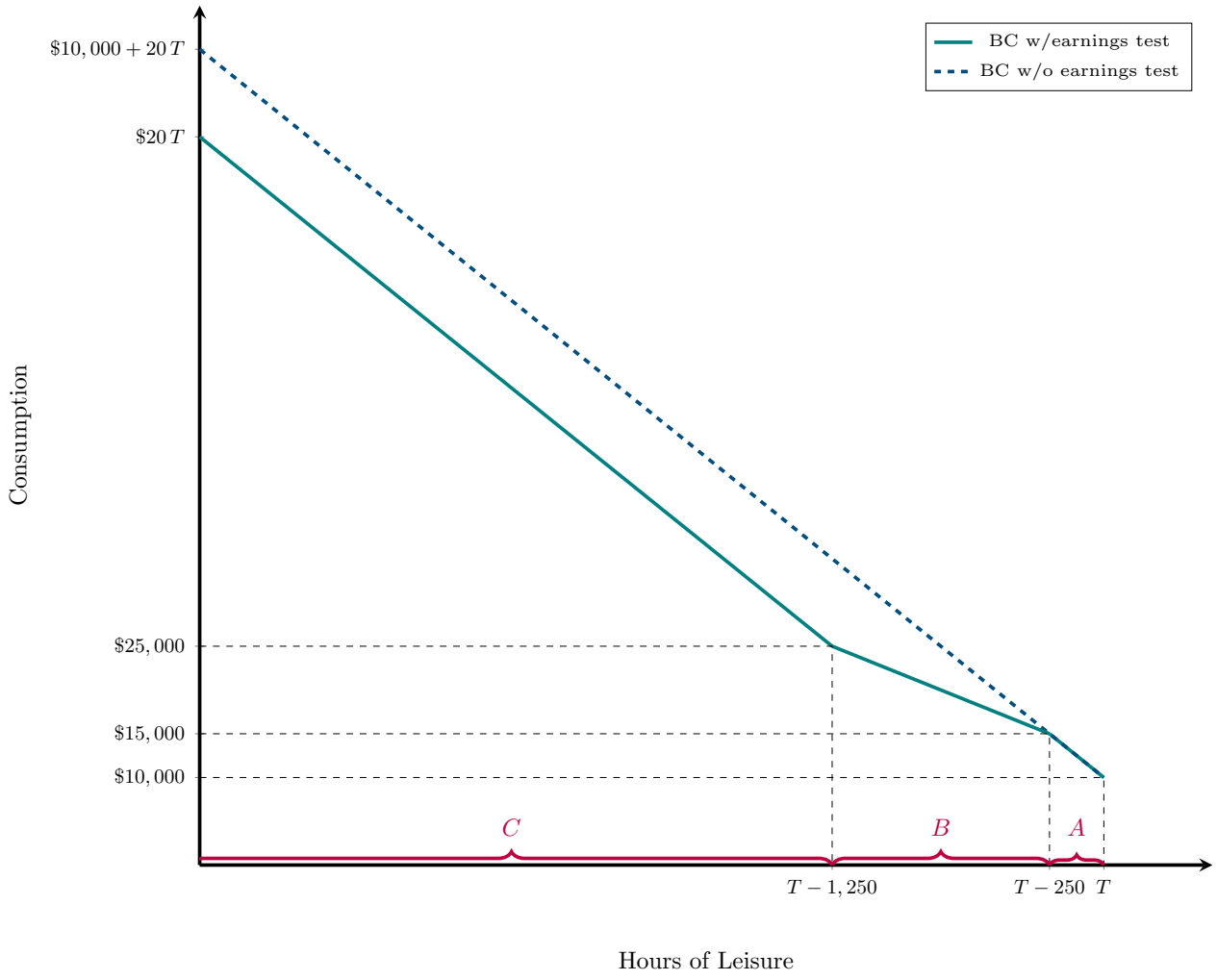
$$C = \begin{cases} 10,000 + 20\,T - 20\,L & \text{if } \; L \in [T - 250,\, T] \\ 12,500 + 10\,T - 10\,L & \text{if } \; L \in [T - 1,250,\, T - 250) \\ 20\,T - 20\,L & \text{if } \; L \in [0,\, T - 1,250)\,, \end{cases}$$

   In contrast, if there were no earnings test, nonlabour income would be constant and equal to b, so the budget line would be

$$C = 10,000 + 20\,T - 20\,L.$$

   Both budget lines are plotted in Figure 3 below.

Figure 3: Budget line with and without earnings test



Explanation: if the person works 250 hours he/she earns 250 hours × \$20 per hour = \$5,000 so that's the maximum they can earn and still keep all their \$10,000 benefit for a total of \$15,000. Beyond that point the benefit is gradually phased out. If they work a further 1,000 hours (for a total of 1,250) the benefits phased out are 1,000 hours × \$20 per hour × 0.5 phase out = \$10,000. So at that point all the benefits are fully phased out and working more gives the full wage again.

(b) You can distinguish three regions on your budget constraint for a worker subject to the earnings test. What would be the labour supply response of retirees who choose hours in each of these regions if the earnings test were eliminated? What would be the change in labour force participation?

Answer:

The three regions are labelled A, B, and C in Figure 3 above. The possible cases, illustrated in Figure 4 bellow, are as follows.

(A) In region A, below the floor, workers are unaffected by the earnings test; they are not

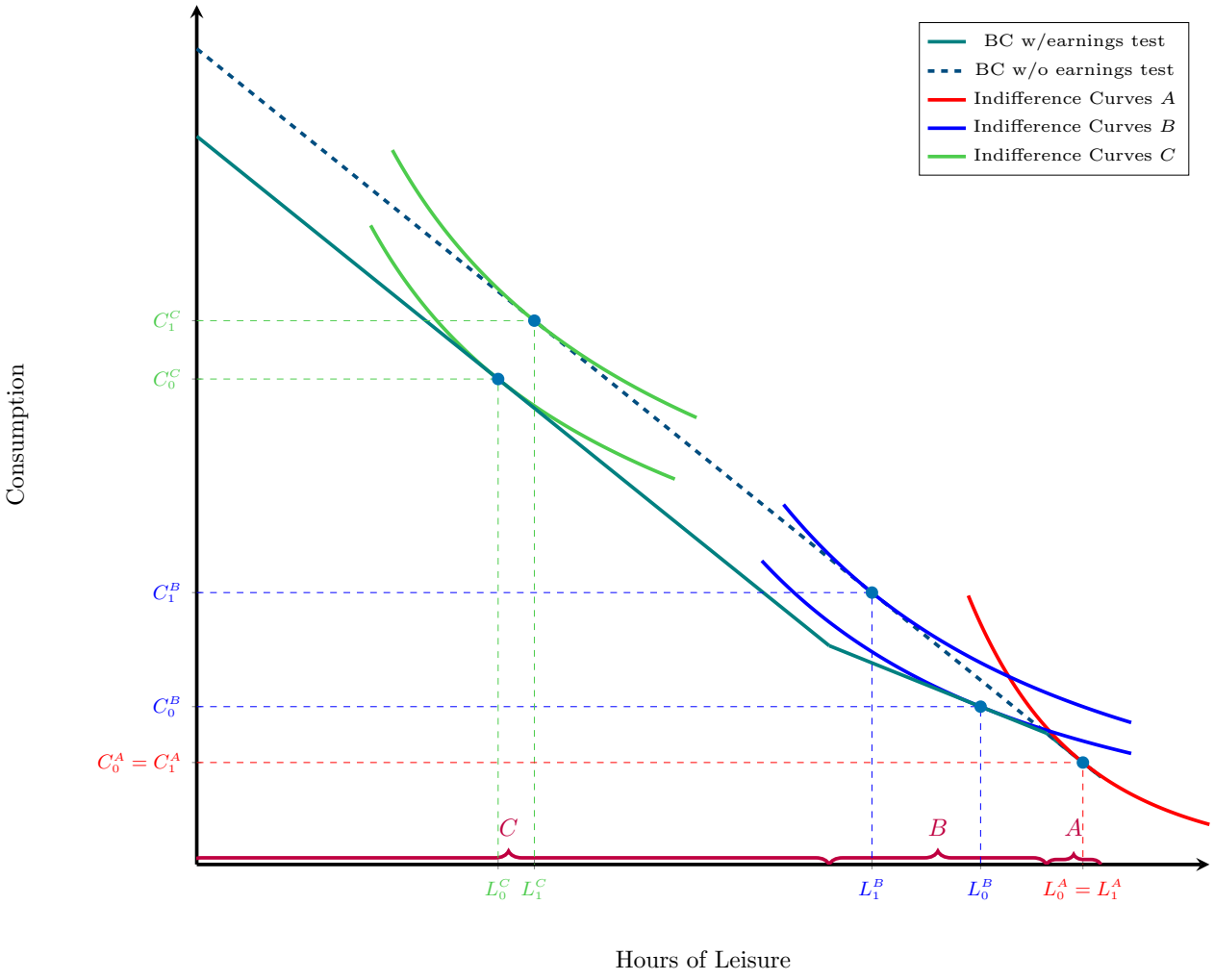
going to change their behaviour when the floor is eliminated.

(B) In region B, where the earnings test imposes a 50% tax on earnings, eliminating the test would be equivalent to an increase in wages. This will have an income and substitution effect and therefore an ambiguous effect on work hours.

(C) In region C, where all Social Security benefits are taxed away, elimination of the earnings test does not change the slope of the budget constraint but only introduces an income effect. Therefore, workers in this region will tend to work less.

The total change in hours is indeterminate. There should be no change in participation. Not working at full benefits was a choice that was available before, so elimination of the earnings test should not induce anyone to stop working. The change should also not induce anyone to start working. The only new segment of the budget constraint after the change is above the floor. It is impossible to draw convex indifference curves so that one is at the corner of not working and a higher indifference curve is tangent on the new segment above the floor.

Figure 4: Choices with and without earnings test



(c) Suppose the earnings test floor is raised from \$5,000 to \$10,000. What are the possible labour supply responses of retirees to this change? How many separate regions of the budget constraint do you have to consider for the analysis now?

Answer:

By the same reasoning of part (a), nonlabour income is given by

$$V(wH) = \begin{cases} 10,000 & \text{if } \; 20H \in [0,\, 10,000] \iff H \in [0,\, 500] \\ 15,000 - 10H & \text{if } \; 20H \in (10,000,\, 30,000] \iff H \in (500,\, 1,500] \\ 0 & \text{if } \; 20H \in (30,000,\, 20\,T] \iff H \in (1,500,\, T]\,. \end{cases}$$

Thus, the budget constraint is given by

$$C = \begin{cases} 10,000 + 20\,T - 20\,L & \text{if } \; L \in [T - 500,\, T] \\ 15,000 + 10\,T - 10\,L & \text{if } \; L \in [T - 1,500,\, T - 500) \\ 20\,T - 20\,L & \text{if } \; L \in [0,\, T - 1,500)\,. \end{cases}$$

Figure 5 shows the budget constraint for both earnings test floors. There are five regions now:

(A) Again, there is no response in region A.

(B) In region B, there is an income and a substitution effect of opposite signs. The hours effect is indeterminate.

(C) In region C, there is a pure income effect $\implies$ lower hours.

(D) In region D, the wage has decreased but income is higher. Income and substitution effect go in the same direction $\implies$ lower hours.

(E) From region E, we might get people to opt into regions C or D $\implies$ lower hours.

Figure 6 below illustrates these different scenarios.

Figure 5: Budget line with $5,000 and $10,000 floors

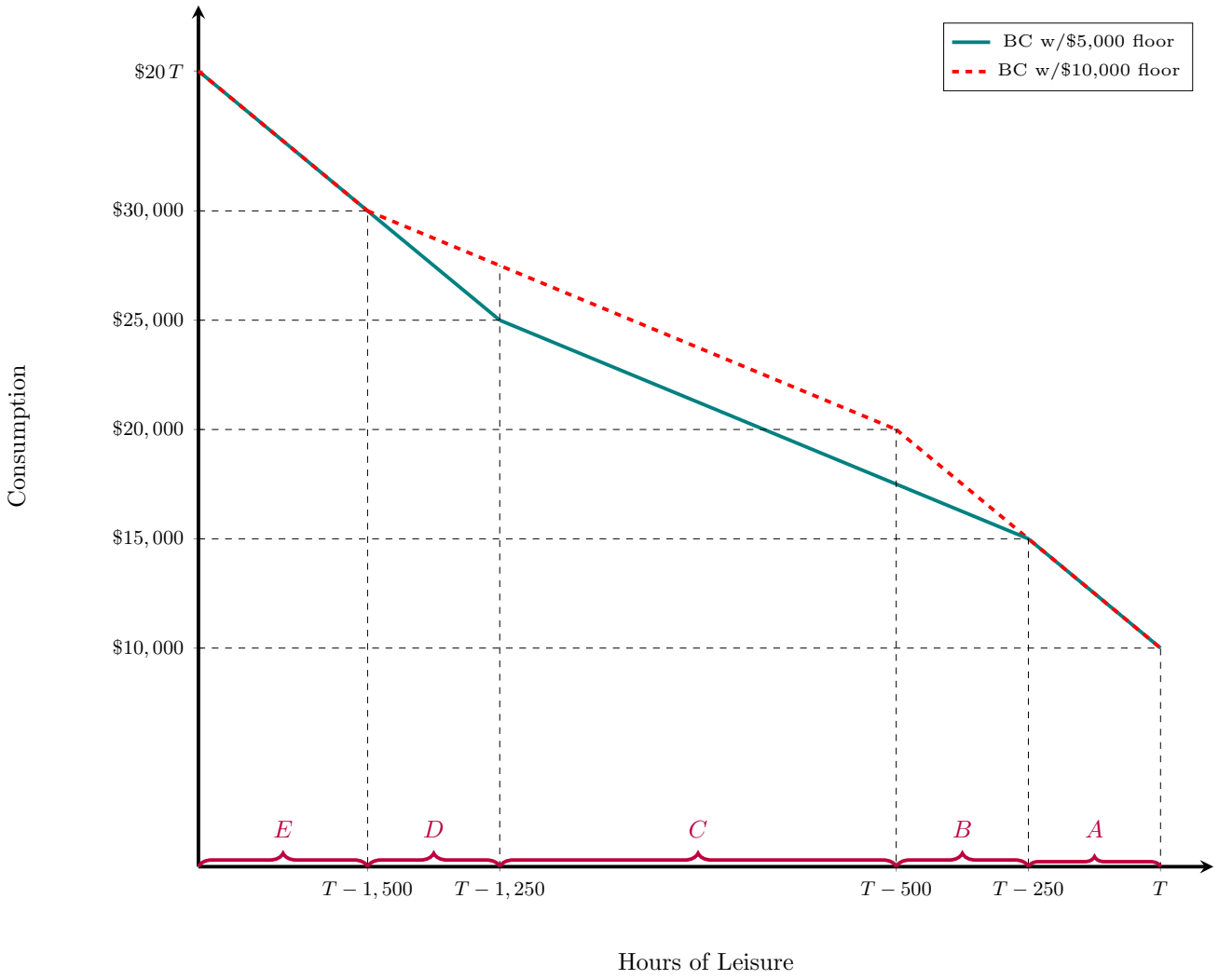


Figure 6: Choices under $5,000 and $10,000 floors